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Abstract.
We analyze the social optimality of growth and product variety in a model of endogenous growth. The model contains two sectors, one assembly sector producing a homogenous consumption good, and one intermediate goods sector producing a differentiated input used in the assembly sector. Growth results from R&D performed by firms in the intermediate goods sector aimed at quality improvement. We disentangle three effects associated with increased variety, namely (i) a productivity effect, (ii) a business stealing effect, and (iii) a growth effect. The market provides too little variety and suboptimally high growth if the productivity effect of variety is large relative to the market power of intermediate goods producers. If varieties are not productive, the market provides too low a rate of growth, whereas variety may be too low as well.

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We analyze the social optimality of growth and product variety in a model of endogenous growth. The model contains two sectors, one assembly sector producing a homogenous consumption good, and one intermediate goods sector producing a differentiated input used in the assembly sector. Growth results from R&D performed by firms in the intermediate goods sector aimed at quality improvement. We disentangle three effects associated with increased variety, namely (i) a productivity effect, (ii) a business stealing effect, and (iii) a growth effect. The market provides too little variety and suboptimally high growth if the productivity effect of variety is large relative to the market power of intermediate goods producers. If varieties are not productive, the market provides too low a rate of growth, whereas variety may be too low as well.

1. Introduction.

Product variety is an important determinant of economic welfare. Following the seminal work by Dixit and Stiglitz (1977), and Spence (1976) the welfare effects of variety have been analyzed from various angles.[1] Dixit and Stiglitz themselves conceive the problem of optimal diversity (in a static context) as one of quantity versus diversity. With the presence of economies of scale in production, producing a small variety saves resources that can be used to extend the production volume. Hence a trade-off arises that gives rise to the question of social optimality of the market equilibrium. It turns out that the market supports too low product diversity. Subsequent studies addressed the optimality question in the presence of growth. In a dynamic context, reduced variety not only saves resources that can be used for extending the produced quantity, but potentially also to increase the rate of growth. Grossman and Helpman (1991, chapter 3) analyze welfare in a model of endogenous growth. In their analysis, there is (continuous) growth in product variety resulting from investment in R&D. The more labour an economy allocates in the R&D

[1] In the overview to follow, we have no pretention of being exhaustive.

sector, the less labour remains for producing consumption goods. The question here is one of growth in variety versus volume of consumption goods. The optimal trajectory entails more rapid growth of variety than the market equilibrium sustains, as firms ignore the contribution of their knowledge creation to a common 'knowledge pool'. Grossman and Helpman (1991) also analyze a quality ladder model. In this model quality is endogenous and variety is exogenous. Here innovative effort aimed at quality improvement might be suboptimal high or low, depending on the size of the quality step. Van de Klundert and Smulders (1997) develop an endogenous growth model in which, contrary to Grossman and Helpman (1991), R&D is an in-house activity aimed at improving quality. Besides quality growth, variety is also determined endogenously. An important characteristic of the model is the constancy of variety in equilibrium. The authors analyze the welfare consequences of different regimes of competition in the presence of knowledge spill-overs. They find suboptimally low growth. Product variety may both be too small and too large.

The studies discussed so far assume that variety has a direct effect on consumers' welfare as consumers have a love for variety. Another branch of literature looks at the productivity effects of increased product variety (Ethier, 1982). Romer (1990) takes this route and develops a model in which diversity of capital inputs grows at a constant rate, increasing productivity at a constant rate. In a similar spirit, the focus in the underlying paper will be on the effects of diversity on productivity. Barro and Sala-i-Martin (1995, chapter 6) point at the similarity of endogenous growth models of on the one hand expanding variety of consumption goods directly adding to utility, and the models of expanding variety of producer goods increasing productivity and indirectly adding to utility on the other hand. Ultimately, it is the flow of utility that matters. Still, they strongly argue in favour of expanding product variety models that add to productivity as they can more easily be reconciled with reality.

This paper develops a simple two sector model with endogenous growth that sheds some light on the advantages and disadvantages of using variety of differentiated inputs. Our starting point is the model developed by Van de Klundert and Smulders (1997). The strength of this model in view of the topic we want to study is that it is characterized by *both* endogenous growth and endogenous product variety (Peretto (1996) develops a similar kind of model). Firms in the assembly sector of the model produce a homogeneous consumption good. Production takes place using (homogeneous) labour that is supplied inelastically, and differentiated intermediate goods. We follow Ethier (1982) in assuming that there are returns to variety. Ethier explicitly singles out the returns to variety (that are external to the firms producing the brands), and the elasticity of substitution between brands. This is in contrast with, *e.g.*, Dixit and Stiglitz (1977), and Van de Klundert and Smulders (1997). In these studies returns to variety and the elasticity of substitution between brands are mechanically linked. Although the return to variety and the elasticity of substitution are likely to be (negatively) related, there is no reason to assume that the link is as tight as suggested in the earlier mentioned papers. As we will show in this paper, disentangling this relation is crucial for the welfare effects that we derive. A similar point in a different context is made by, *e.g.*, Benassy (1996), and Broer and Heijdra (1996). The intermediates are produced in the second sector of the model. In this sector, firms are producing a unique brand of an intermediate good. Labour is employed in each firm for production and research. Intentional investments in R&D aimed at increasing productivity in this sector drive economic growth. R&D is modelled as an in-house activity yielding completely firm-specific knowledge. Hence, the relevant knowledge base for R&D is the stock of knowledge that is built up by the firm's own past research activities. The uniqueness of brands gives rise to market power. The resulting non--competitive pricing of differentiated products results in a distortion as it affects the relative price of inputs in production of the final consumption good. Two other potentially distorting market failures are present. The first arises because entrepreneurs do not take into account the surplus of their entry decision that accrues to the producers of the consumption good, due to increasing returns to variety. The second can be traced to the fact that entrants ignore that entry implies that the total market has to be shared by more. The first will be dubbed the *product diversity effect* whereas the second will be called, following Mankiw and Whinston (1986), the *business stealing effect*.[2]

We show that explicitly separating out these three potential distortions is crucial for understanding the results of the welfare analysis. Crucial is the size of the product diversity effect relative to the elasticty of substitution. The market results in too high growth and a suboptimally low level of variety when the product diversity effect is strong. If the business stealing effect dominates (and hence the returns to variety are relatively

[2] The notion of business stealing has strong resemblance to the *profit destruction effect* that Grossman and Helpman (1991) distinguish.

low), the market growth is too low. If the diversity effect is extremely weak, the variety supplied will be suboptimally high. If it is extremely strong, the opposite holds. In the intermediate case, it may be the case that *both* the rate of growth and variety are too low.

The paper proceeds as follows. In section 2, we will discuss and present the basic model. The steady state equilibrium of the market economy is presented in section 3. In section 4, we perform a first best analysis by looking at the unconstrained social optimum, and we compare this with the market outcome. Furthermore, in section 5, we consider the second best problem of choosing the welfare maximizing number of firms (by issuing permits), taking as given the non-competitive behaviour after entry. We end with conclusions and an evaluation in section 6.

2. The model

Our economy comprises two sectors. The assembly sector produces a homogeneous consumption good using intermediates and labour. Firms in this sector operate under perfect competition and take prices of intermediates and wages as given. The intermediates are imperfect substitutes in production of the homogeneous consumption goods. Following Ethier (1982), there is an externality in this sector in the form of increasing returns to variety (productivity increases with variety). In the intermediate goods sector, N firms are operating. These firms compete monopolistically. We assume N to be sufficiently large so that competition is monopolistically à la Chamberlin.[3] The number of intermediate goods producers is determined endogenously by a process of entry or exit as long as profits are non-zero. Intentional R&D performed by firms in this sector results in productivity increases and thus positively affects the quality of intermediates. We assume R&D to be an in-house activity. There is no spill-over of the fruits of R&D whatsoever, *i.e.*, we assume knowledge to be fully tacit. In this section, we will in turn describe consumer

[3] Yang and Heijdra (1993) criticize this approach. They argue that the result that the number of firms is so large that competition is monopolistically à la Chamberlin should be the *outcome* of the model and not be imposed à priori. To avoid this problem, we would have to introduce the concept of a perceived price elasticity, where this perceived elasticity depends on the number of competitors (see Van de Klundert and Smulders (1997)). As this extension would seriously complicate the analysis and not affect the main conclusions of the paper, we abstain from this issue.

behaviour, the assembly sector and the intermediate goods sector.

2.1 Consumer behaviour

A representative consumer maximizes his intertemporal utility subject to a dynamic budget constraint

$$\max \int_{o}^{\infty} u(C_t) e^{-\theta t} dt \quad s.t. \quad \dot{A}_t = r_t A_t + w_t L_t - C_t P_{Ct}, \tag{1}$$

where C_t is a consumption index, θ is the subjective discount rate, A_t are assets possessed by consumers,[4] r_t is the interest rate, $w_t L_t$ is wage income in the economy, and $C_t P_{Ct}$ is expenditures on consumption goods. We assume a constant working population L. Taking $u(C_t) = C_t^{1-\rho}/(1-\rho)$, we arrive at the Ramsey rule

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\rho}\left(r_t - \frac{\dot{P}_{Ct}}{P_{Ct}} - \theta \right), \tag{2}$$

where $1/\rho$ is the intertemporal elasticity of substitution. So consumers prefer a steeper consumption profile, the larger the gap between the real rate of interest (r_t-$\dot{P}_{Ct}/P_{Ct}$) and the subjective discount rate (θ), and the larger the intertemporal elasticity of substitution ($1/\rho$).

2.2 Assembly sector.

The assembly sector produces final consumption goods C under perfect competition. The goods are produced according to (dropping time indices where it leads to no confusion)

$$C = X^{\beta} L_C^{1-\beta} \quad \text{with} \quad X = N^{\sigma} \left[\frac{1}{N} \sum_{i=1}^{N} x_i^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{3}$$

In this formulation, L_C represents the number of production workers in the assembly sector. X is a composite of the intermediates that are available. This specification of the

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[4] The assets A consist out of consumer loans and shares issued by high-tech firms to finance their investments in research. As in a consolidated equilibrium, net debt among consumers is zero, the income on the assets, rA, equals the dividends paid by high-tech firms. We will later return to the savings-investment equilibrium when discussing the general equilibrium of the model.

composite is borrowed from Broer and Heijdra (1996). N represents the number of varieties of the intermediate good (indexed i) available. ε is the elasticity of substitution between any pair of differentiated intermediates. Returns to variety are represented by the parameter σ. With all the x_i equal to a common x, as will be the case in equilibrium, we get $X=N^{\sigma-1}(Nx)$. Suppose that there are two bundles of intermediates that are equally large ($N^1x^1=N^2x^2$). It then holds that if $\sigma>1$ (that are returns to variety), the bundle with the largest variety (the largest N) is most productive. Parameter restrictions $\sigma\geq1$ and $\varepsilon>1$ ensure that respectively production exhibits returns to variety (with strict inequality) and that every variety is demanded. The returns to variety equal the special value assumed by Dixit and Stiglitz (1977) if $\sigma=\varepsilon/(\varepsilon-1)$. The composite of intermediates specified in equation (3), can hence be seen as a generalization of Dixit and Stiglitz (1977).[5]

The producers take prices for intermediates, output prices and wages as given and perform the following (two stage) maximization problem

$$\max_{L_C,\, X} \; CP_C - wL_C - P_x X, \tag{4}$$

where P_X is the price index for the composite good.

Optimization yields

$$\frac{\partial\Pi}{\partial X} = 0 \;\Leftrightarrow\; \beta CP_C = XP_X, \text{ and} \tag{5}$$

$$\frac{\partial\Pi}{\partial L_C} = 0 \;\Leftrightarrow\; (1-\beta)CP_C = L_C w, \tag{6}$$

which is the standard Cobb-Douglas result of fixed income shares.

In the second stage, the firm decides on the optimal amount of input of each

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[5] It is easily demonstrated that in case $\sigma=\varepsilon/(\varepsilon-1)$, the expression for the composite good boils down to

$$X = \left[\sum_{i=1}^{N} x_i^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}, \text{ in symmetry } X = N^{\frac{1}{\varepsilon-1}}(Nx).$$

This specification is used in many subsequent analyses (*e.g.*, Grossman and Helpman (1991) and Van de Klundert and Smulders (1997)). We prefer the more general Ethier specification as we see no need for a strict one to one relation between the mark-up resulting from market power and the returns to diversity. See Benassy (1996) for a similar argument.

variety. The following optimization problem is solved

$$\underset{x_i}{\text{Max}} \; N^{\sigma}\left[\frac{1}{N}\sum_{i=1}^{N} x_i^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{s.t.} \quad \sum_{i=1}^{N} x_i p_{xi} = XP_X, \tag{7}$$

which results in

$$x_i = N^{(\varepsilon-1)(\sigma-1)-1}\left[\frac{p_{xi}}{P_X}\right]^{-\varepsilon} X, \text{ where } P_X \equiv \frac{\sum_{i=1}^{N} x_i p_{xi}}{X} = N^{1-\sigma}\left[\frac{1}{N}\sum_{i=1}^{N} p_{xi}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \tag{8}$$

which is the downward sloping demand curve for an intermediate of brand *i*. It is easily seen that under symmetry the true price index ($P_X=N^{1-\sigma}p_x$) is decreasing in N if $\sigma>1$.

2.3 The Intermediate Goods sector.

The intermediate goods sector consists out of N firms, each producing a brand of a differentiated input used in the assembly sector. Firm i produces a quality adjusted amount x_i of the intermediate of type/brand i according to

$$x_i = h_i L_{xi}, \tag{9}$$

where L_{xi} represents production labour in the intermediate goods sector, and h_i is the labour productivity. R&D is aimed at quality innovation. The stock of knowledge, which is assumed to be completely firm-specific, determines the quality level directly. Knowledge accumulates according to

$$\dot{h}_i = \xi h_i L_{ri}, \tag{10}$$

where L_{ri} represents research labour and ξ is the research productivity parameter. The firms aim at maximizing their present discounted value subject to the demand for intermediates and the costs and benefits of engaging in R&D. The objective can hence be written as

$$\max \int_0^\infty \left[x_i p_{xi} - (L_{xi}+L_{ri}+L_f)w\right] e^{-rt} dt. \tag{11}$$

L_f is fixed labour required for starting production, and r is the interest rate at which firms could invest their money in the financial market.

The current value Hamiltonian corresponding to this dynamic optimization problem reads as

$$H = x_i p_{xi} - (L_{xi}+L_{ri}+L_f)w + p_{hi}\xi h_i L_{ri}, \tag{12}$$

where p_{hi} is the shadow price corresponding to knowledge.

The First Order Conditions to the intertemporal optimization problem are (assuming symmetry so that we can drop the brand index i)

$$\frac{\partial H}{\partial L_x} = hp_x(1 - \frac{1}{\varepsilon}) - w = 0 \;\; \Leftrightarrow \;\; p_x = \frac{\varepsilon}{\varepsilon - 1}\frac{w}{h}, \tag{13}$$

according to which firms engage in mark-up pricing (note that the mark up ($\varepsilon/(\varepsilon$-1)) reflects the increasing returns to variety in case $\sigma=\varepsilon/(\varepsilon$-1), see footnote 5),

$$\frac{\partial H}{\partial L_r} = -w + p_h \xi h = 0 \;\; \Leftrightarrow \;\; w = p_h \xi h, \tag{14}$$

showing that firms allocate R&D labour to this sector as long as the marginal benefits of doing so ($p_h\xi h$) exceed the marginal costs (w), and

$$-L_x p_x \frac{\varepsilon - 1}{\varepsilon} - p_h \xi L_r = \dot{p}_h - rp_h \;\; \Leftrightarrow \;\; \frac{\dot{p}_h}{p_h} + L_x \frac{p_x}{p_h}\frac{\varepsilon - 1}{\varepsilon} + \xi L_r = r, \tag{15}$$

which is the no arbitrage condition. Investing an amount p_h in the financial market at the rate r should yield the same return as investing in knowledge capital which yields a capital gain, an increase in production, and an increase in the knowledge base.

The model is closed by imposing full employment labour market equilibrium

$$L = L_C + N(L_x+L_r+L_f), \tag{16}$$

and imposing instanteneous profits in the intermediate goods sector (π) to equal zero

$$\pi \;=\; xp_x - (L_x+L_r+L_f)w \;=\; 0\,. \tag{17}$$

Characterization of the steady state solution of the model will be the topic of the next section.

3. Solution of the model

In this section, we characterize the *steady state* solution of the model and discuss its main comparative statics characteristics. We assume that excess profits (or losses) are competed away by free entry and/or exit of firms (ignoring integer constraints). The number of intermediate goods producers is thus determined endogenously. We define the growth rate of labour productivity in the intermediate goods sector as $g(\equiv\dot{h}/h)$. In the remainder of the paper, we take the wage rate as numéraire (w=1). The derivations of the expressions for the equilibrium rate of growth, the equilibrium interest rate and the equilibrium number of firms are given in Appendix A.

Solving the model (see Appendix A) yields a required and planned rate of return on savings and investment, respectively

$$r \;=\; \theta + \beta(\rho-1)g \quad \text{and} \quad r \;=\; (\varepsilon-1)(g+\xi L_f)\,. \tag{18}$$

The required real rate of rate of return on savings (r-$\dot{P}_C/P_C$=r+βg) thus increases with the rate of growth of consumption (βg) due to the wish of consumers to smooth their consumption over time (ρ>0). The realized rate of return depends positively on the rate of growth, the productivity of research and the fixed cost requirement. Confronting the realized and required rate of return results in a savings-investment equilibrium and yields the equilibrium growth and interest rate[6]

$$g \;=\; \frac{(\varepsilon-1)\xi L_f-\theta}{\beta(\rho-1)-(\varepsilon-1)} \quad \text{and} \quad r \;=\; (\varepsilon-1)\frac{\beta(\rho-1)\xi L_f-\theta}{\beta(\rho-1)-(\varepsilon-1)}\,. \tag{19}$$

6 Stability of the equilibrium with a positive growth rate requires $\beta(\rho\text{-}1)>(\varepsilon\text{-}1)>\theta/(\xi L_f)$.

Finally, we derive the equilibrium number of intermediate goods producers (the variety of intermediates) and the allocation of labour as

$$N = \frac{[\beta(\rho-1) - (\varepsilon-1)]\xi\beta L}{\varepsilon[\beta(\rho-1)\xi L_f - \theta]} \quad \text{and} \quad L_C = (1-\beta)L. \tag{20}$$

An important feature of the model is that the scale of the economy (L) leaves the growth rate unaffected (see Van de Klundert and Smulders (1997)), whereas it leads to an equiproportionate change in the number of firms. Important here is the notion that an increase in the size of the economy leaves individual firm size unaffected and thus leaves the incentive to engage in R&D unchanged. Scale effects are thus only present in the sense that productivity *levels* are affected.[7] Another important notion is that the parameter capturing the 'returns to diversity' does not show up in the solution for the equilibrium variety. The market does not take into account the externality resulting from diversity.

For the growth rate, we can conclude that it positively depends on the fixed cost requirement L_f, the elasticity of substitution ε, and the research productivity parameter ξ. An increase in the fixed cost requirement lowers the equilibrium number of firms and increases the market share of each individual firm. This increases the incentive to engage in R&D and consequently the growth rate. A similar type of argument holds for the elasticity of substitution. A large elasticity of substitution lowers the room for firms to make positive profits and thus increases individual firm size. An increase in θ and ρ reduces the consumers' incentive to save and thus increases the costs of acquiring financial means to finance investment in knowledge capital. Firms will respond by investing less in knowledge capital, reducing the rate of growth (and the equilibrium interest rate). The reduction in the investment burden required to keep up with competitors increases the firm's profits, which will lead to entry. An increase in β (the share of intermediates in the production of final consumption goods) increases the required rate of return on savings (as consumption growth increases for a given level of productivity

[7] Labour productivity in the assembly sector (C/L_c) *grows* at rate βg. The labour productivity *level* equals

$$\frac{C}{L_C} = \left(N^{\sigma-1}h\frac{\beta}{1-\beta}\frac{(\varepsilon-1)}{\varepsilon}\right)^{\beta},$$

which positively depends on N (and thus on L) as $\sigma>1$. This result basically reflects Adam Smith's notion of division of labour.

growth). The equilibrium interest and growth rate consequently go down. This will be accompanied by downscaling of intermediate goods producers of both their research and production departments. At the same time, the assembly sector will substitute away in its production process from labour towards intermediates (L_C/NL_x decreases). This combination of reduced high-tech firm size and increased demand for intermediates implies an increase in the number of intermediate firms.

4. Optimal product variety

In this section, we will derive the first best social optimum (FBSO) by solving the social planners problem. Thus the social planner maximizes intertemporal utility of the representative agent solely subject to the technical constraints (*i.e.*, technology and resource availability). Characterization of the social optimum will be the topic of section 4.1. In section 4.2, we will compare the market equilibrium with the social optimum. The results derived under the Dixit-Stiglitz assumption that $\sigma=\varepsilon/(\varepsilon-1)$ turn out not to generalize when dropping this assumption. This points at the importance of disentangling the returns to diversity and the degree of imperfect substitutability resulting in market power of monopolistic competitors (see also Benassy (1996), and Broer and Heijdra (1996)).

4.1. The First Best Social Optimum

In this section, we will look at the first-best optimum in which a social planner is assumed to maximize the utility of the representative agent subject to the accumulation function of firm specific knowledge, the production technology,[8] and the labour market constraint

[8] The model allows for a $\sigma\geq1$ restriction, but with equality the social planners' problem is not well defined. So here the parameter restriction needs to be narrowed to $\sigma>1$. It is intuitively clear that it is socially optimal to have a single variety in case $\sigma=1$. As knowledge is completely firm specific and there is no return to variety as such, it is optimal to minimize on the total fixed cost in the economy. To allow for $\sigma=1$, the restriction $N\geq1$ should be added to the social planners optimization program.

$$\max U = \int_0^\infty \frac{C^{1-\rho}}{1-\rho} e^{-\theta t} dt, \tag{21}$$

subject to equations (3), (10), and (16). The current value Hamiltonian corresponding to this optimal control problem is[9]

$$H = \frac{1}{1-\rho}\left[\left(L - N(L_x + L_r + L_f)\right)^{1-\beta}\left(N^\sigma h L_x\right)^\beta\right]^{1-\rho} + p_h \xi h L_r. \tag{22}$$

The First Order Conditions corresponding to this problem are

$$\frac{\partial H}{\partial L_x} = 0 \Leftrightarrow (1-\beta) N L_x = \beta L_C, \tag{23}$$

$$\frac{\partial H}{\partial L_r} = 0 \Leftrightarrow p_h = (1-\beta)\frac{C^{1-\rho} N}{L_C \xi h}, \tag{24}$$

$$\frac{\partial H}{\partial N} = 0 \Leftrightarrow L_C = \frac{(1-\beta)L}{\beta\sigma + 1 - \beta} = L_C^{FB}, \tag{25}$$

$$\dot{p}_h + \frac{\partial H}{\partial h} = \theta p_h \Leftrightarrow \dot{p}_h + \frac{C^{1-\rho}\beta}{h} + p_h \xi L_r = \theta p_h. \tag{26}$$

From equation (25), it is evident that employment in the assembly sector is constant. Using equation (23), it follows that NL_x is constant. Substitution of the expressions for NL_x and L_C in the labour market constraint (equation (16)) gives an expression for L_r and consequently for the optimal growth rate

$$g^{FB} = \xi L_r^{FB} = \frac{\xi\beta(\sigma-1)}{\beta(\sigma-1)+1}\frac{L}{N^{FB}} - \xi L_f. \tag{27}$$

The optimal growth rate thus negatively depends on the number of intermediate goods available in the economy. The optimal number of intermediate goods producers can be

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9 Use that under symmetry $X = N^{\sigma-1} N x = N^\sigma h L_x$.

derived as[10]

$$N^{FB} = \frac{\beta(\sigma-1)(\rho-1)-1}{\beta(\rho-1)\xi L_f - \theta}\frac{\beta\xi L}{\beta(\sigma-1)+1} \quad \text{so} \quad g^{FB} = \frac{\xi L_f - \theta(\sigma-1)}{\beta(\sigma-1)(\rho-1)-1}. \tag{28}$$

We can derive

$$\frac{\partial N^{FB}}{\partial\sigma} > 0;\ \frac{\partial N^{FB}}{\partial\theta} > 0;\ \frac{\partial N^{FB}}{\partial L_f} < 0;\ \frac{\partial N^{FB}}{\partial\xi} < 0;\ \frac{\partial N^{FB}}{\partial L} > 0;\ \frac{\partial N^{FB}}{\partial\rho} > 0;\ \frac{\partial N^{FB}}{\partial\beta} > 0. \tag{29}$$

An increase in the returns to variety positively affects the optimal number of differentiated inputs, which is an intuitively clear and important result. An increase in the subjective discount rate increases the optimal number of varieties of the intermediate good. The intuition behind this result is as follows. A higher discount rate increases the value attached to current production. This higher production can be achieved by increasing the number of varieties used in the assembly sector, as the positive productivity effect of an additional variety outweighs the negative growth effect that results from increased fixed cost in the economy. Increases in the fixed cost decrease the optimal product variety. The increased fixed cost tends to lower the optimal rate of growth (if N^{FB} is kept constant). This negative effect can partly be offset by decreasing the fixed cost requirement by scaling down the number of product varieties. The other comparative statics results need no further discussion.

The above results make clear that the social planner faces a trade-off between on the one hand large variety with positive productivity effects in the assembly sector, and on the other hand high growth rates in the intermediate goods sector. The larger the returns to variety (the larger σ), the more growth the social planner will sacrifice in exchange for large variety.

4.2 Market equilibrium versus the social optimum

We are now ready to compare the market outcome as discussed in section 3 with the FBSO. The model is characterized by three potentially distorting market failures. Firstly,

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[10] We impose the parameter restriction $1/(\beta(\rho-1))<\sigma-1<\xi L_f/\theta$. Derivation of the optimal number of intermediate goods producers can be found in Appendix B.

there is a (static) distortion in allocation resulting from the market power that intermediate goods producers have, leading them to engage in mark-up pricing. The second distortion results from the fact that entrants in the intermediate goods sector ignore the productivity effect on the producers of consumer goods (the diversity effect). And thirdly, entrants do not take into account that they decrease the effective market size for their competitors (the business stealing effect).

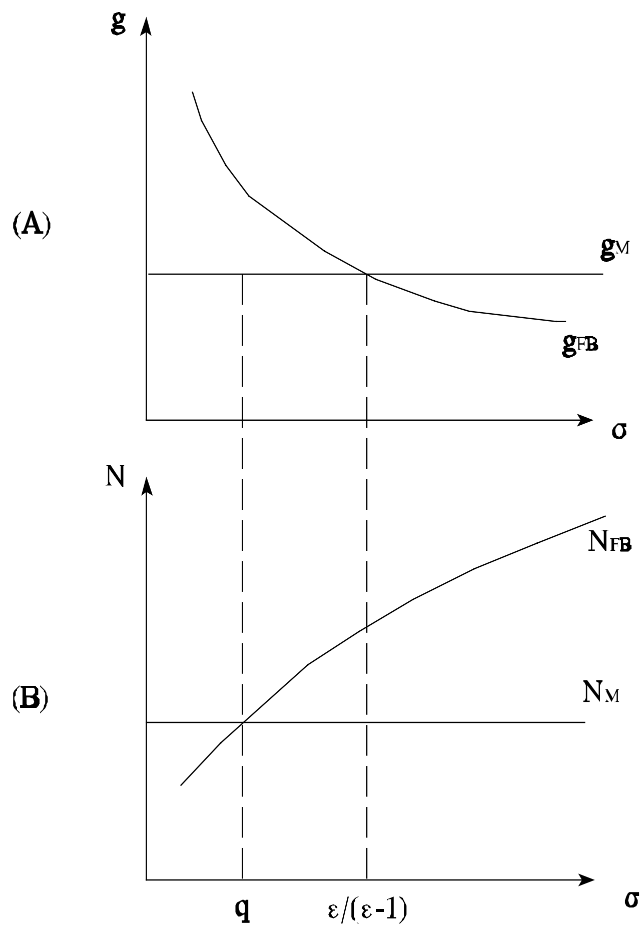

Figure 1. The Market Equilibrium versus the Social Optimum

Figure 1 depicts the comparison between the market equilibrium and the social optimum. Panel (A) depicts the growth rate in the market equilibrium (g^M) and in the social optimum (g^{FB}), whereas Panel (B) depicts the number of firms (product diversity) in the market equilibrium (N^M) and in the social optimum (N^{FB}). The Figure is constructed using equations (19), (20), and (28).[11]

With respect to the rate of growth, we conclude that the market rate of growth is optimal at $\sigma=\varepsilon/(\varepsilon-1)$. This is the specific value for the preference for diversity implicitly assumed by Dixit and Stiglitz (1977). The number of product varieties supplied by the market (N^M) is suboptimally low at this point, which basically restates the Dixit-Stiglitz result. Market power of the intermediate goods producers results in non-competitive pricing whereas labour can be hired in a perfectly competitive labour market. Therefore, in the assembly sector, the use of inputs is distorted into the direction of labour (L_C/NL_x is too high from a social point of view). The too low aggregate demand for intermediates results in too little variety of intermediates. The growth rate is at the socially optimal level in the market economy, as in this special case the elasticity of demand *both* reflects excess entry due to business stealing *and* the contribution of variety to overall productivity.[12] Hence, the product diversity effect and the business stealing effect happen to be of equal magnitude (*cf.* Grossman and Helpman (1991, Appendix A.3.3)).[13]

To the right of the intersection of g^M and g^{FB}, the increasing returns to variety are strong but *not* (fully) reflected in the market by a high mark-up (as in the special case discussed before). The relatively low mark-up means that profit opportunities are gloomy and hence that the market only supports a low number of firms (see panel (B) of Figure 1). Hence the number of firms in the market is suboptimally low. Investing in R&D is a fixed cost that can be spread over more sales in case the market is larger. Hence, the existence of a low number of firms makes the return to R&D high and causes the market to produce a suboptimally high growth rate. Left to the intersection of the curves N^M and N^{FB}, the

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[11] From equations (19), (20), and (28), we can derive $\partial g^M/\partial\sigma=\partial N^M/\partial\sigma=0$, $\partial g^{FB}/\partial\sigma<0$, $\partial^2 g^{FB}/\partial\sigma^2>0$, and $\partial N^{FB}/\partial\sigma>0$, $\partial^2 N^{FB}/\partial\sigma^2<0$. The point of intersection of g^{FB} and g^M is at $\sigma=\varepsilon/(\varepsilon-1)$.The point of intersection of N^{FB} and N^M (point q in Figure 1) is at $\sigma=1+[\beta(\rho-1)+1]/[\beta\{\rho(\varepsilon-\beta)-(1-\beta)\}]$. By straightforward calculation it follows that $1+1/(\beta(\rho-1))<q<\varepsilon/(\varepsilon-1)$.

[12] Another way of putting this is to say that firm *size* is optimal, and the decision to invest in R&D is not distorted in any other way. There is no appropriability problem as there are no spillovers, hence the firm's knowledge base is independent of the number of firms. The private surplus from innovation equals the social surplus.

[13] Decentralizing the social optimum in the special case of $\sigma=\varepsilon/(\varepsilon-1)$ can be achieved by means of the introduction of a subsidy on the use of intermediates. A subsidy is required as market power of intermediate goods producers resulted in too low a demand for intermediates. Achieving the optimal mix of the use of labour relative to the use of intermediates thus requires the introduction of a subsidy s equal to $1/\varepsilon$.

argument presented above can be completely reversed; when the returns to variety are low, the market provides too much variety and too low a rate of growth. In the intermediate case where $q<\sigma<\varepsilon/(\varepsilon-1)$ the market provides too little variety and too low a rate of growth. The intuition for this result can be explained in three steps. (1) In the parameter range under consideration the social return to variety is relatively low. The entry decision, however, is based on the price elasticity (ε) that is relatively low (profit opportunities are relatively high). This suggests a tendency for excessive entry. (2) A countervailing power arises by distortionary pricing of differentiated goods producers. By cost minimization of the assembly firm, this leads to too low aggregate demand for intermediates. This biases the entry decision downward. On balance entry is suboptimally low. (3) As we have seen, non-competitive pricing as such does not impede a Pareto optimal growth rate (see panel (A) at $\sigma=\varepsilon/(\varepsilon-1)$). For the growth we only have to take into account the part of the argumentation under (1).[14] This implies, following the logic phrased above, a suboptimal low rate of growth.

To conclude this section, we have seen that the market growth rate is optimal in the special case when $\sigma=\varepsilon/(\varepsilon-1)$. If the diversity effect is stronger, the market provides a suboptimally high growth rate and vice versa. The number of varieties served by the market is suboptimally low if $\sigma=\varepsilon/(\varepsilon-1)$ which reiterates the Dixit-Stiglitz result of insufficient entry, but now in a dynamic context. If, however, the diversity effect is less pronounced, the market might support a number of varieties that is either too high or too low from a social viewpoint.

5. The optimal number of permits

Successful implementation of industrial policies to achieve the first best results derived in the previous section may be difficult. In this section, we consider the second best problem

[14] This reasoning can be illustrated as follows: if mark-up pricing (leading to too low aggregate demand for intermediates) would evaporate, the distortions on growth and on variety would be balanced again. This is easily demonstrated by setting β equal to one (*i.e.*, no direct labour is used in the production of consumption goods). It then holds that $q=\varepsilon/(\varepsilon-1)$, so the area between q and $\varepsilon/(\varepsilon-1)$ in Figure 1 vanishes. Note the similarity of this version of the model (with $\beta=1$ and $\sigma=\varepsilon/(\varepsilon-1)$) with the model of Grossman and Helpman (1991, chapter 3).

of choosing the welfare maximizing number of firms, taking as given the non-competitive behaviour after entry of intermediate goods producers (this closely resembles the analysis in Mankiw and Whinston (1986), but now in a dynamic context). Hence, it is assumed that the social planner cannot control the behaviour of a given number of firms, but can choose the number of allowances. In order to derive the socially optimal number of permits to be put on the market, we derive an explicit expression for welfare (for the given behaviour of firms).

We recall the utility function introduced in section 2

$$U_0 = \int_0^\infty \frac{C_t^{1-\rho}}{1-\rho} e^{-\theta t} dt. \tag{30}$$

Using

$$C_t = X_t^\beta L_{Ct}^{1-\beta}, \tag{31}$$

and the fact that the allocation of labour and the growth rate are constant over time, we arrive at

$$U_0 = \int_0^\infty \frac{\left[(N^\sigma h_0 L_x)^\beta L_C^{1-\beta} \right]^{1-\rho}}{1-\rho} e^{[(1-\rho)\beta g - \theta]t} dt, \tag{32}$$

where h_0 is the initial productivity level (which subsequently grows at the constant rate g). Integrating this expression finally yields

$$U_0 = \frac{-1}{(\rho-1)\left[\beta g(\rho-1)+\theta\right]\left[(N^\sigma h_0 L_x)^\beta L_C^{1-\beta}\right]^{\rho-1}}. \tag{33}$$

The next step in our analysis is to characterize the effect of a change in the number of product varieties on utility. The sign of the derivative of the present discounted utility w.r.t. N equals

$$\text{sgn.}\left(\frac{dU_0}{dN}\right) = \text{sgn.}\left(\sigma\beta\frac{1}{N} + \beta\frac{1}{L_x}\frac{\partial L_x}{\partial N} + (1-\beta)\frac{1}{L_C}\frac{\partial L_C}{\partial N} + \frac{\beta}{\beta g(\rho-1)+\theta}\frac{\partial g}{\partial N}\right). \tag{34}$$

Utility is affected by an increase in the number of varieties of the intermediates (*i.e.*, the number of permits issued by the government) through four channels[15]:

1. There is a direct *positive* productivity effect (the first term); this is due to the diversity effect in the Ethier specification. This effect is increasing in σ, the returns to diversity parameter, and decreasing in *N*, the level of variety already attained;
2. There is a *negative* volume effect (second term) related to business stealing or profit destruction; more varieties reduce the size of firms producing intermediates and thereby reduce the produced volume of intermediates;
3. There is a *negative* effect on consumption goods production (third term); more varieties are resource consuming by the increase of the total fixed cost in the economy, reducing labour available for production of the consumption good in the assembly sector.
4. There is a *negative* growth effect (the fourth term); more varieties reduce the scale of operations for each firm and hence the profitability of engaging in R&D. The weight for this (negative) effect depends negatively on the pure rate of time preference (θ), and positively on the intertemporal elasticity of substitution ($1/\rho$).

The negative effects are essentially caused by two factors, *viz.*, an effect running via the 'potential of the economy to have large intermediate goods producers with a high growth potential' (L/N), and an effect running via the effective supply of labour ($L\text{-}NL_f$) that is affected by increases in fixed costs following an increase in variety.

The optimal number of firms in the second best social optimum (SBSO) can now be derived as

$$N^{SB} = \frac{\beta(\sigma-1)(\rho-1)-1}{\beta(\rho-1)\xi L_f - \theta} \; \frac{\beta\xi L}{\beta(\sigma-1)+1} \; . \tag{35}$$

Comparing this solution with the solution for the FBSO (equation (28)) reveals that the product variety chosen by the planner who has only one instrument at his disposal is exactly equal to the FBSO. This reflects the fact that the marginal utility of an additional variety is independent of the rate of growth. So comparing the SBSO for the number of

[15] Note that in performing this analysis, we look at an *exogenous* change in *N*. The number of firms is determined exogenously. We assume in other words that entry is blocked (or, alternatively, exit not required). The solution of the model under the assumption that there is no free entry is given in Appendix C.

intermediates with the solution in the decentralized equilibrium gives rise to the same conclusions as described in section 4.2.

In the special case where β=1 (*i.e.*, only intermediates are used in the assembly sector and hence the potentially distorting market failure due to mark-up pricing becomes ineffective as relative prices in the assembly sector are no longer distorted), the second best growth rate equals the first best.[16] So when the distortion in factor allocation resulting from mark-up pricing by intermediate goods producers is 'eliminated', the social planner that has only one instrument at his disposal (the number of permits) can perfectly replicate the FBSO. In the more general case with β<1, the growth rate in the second best is always lower than in first best (see Appendix B). Finally, comparing the growth rate in the SBSO with the market equilibrium basically yields the same qualitative conclusions as in section 4.2. In the special case of $\sigma=\varepsilon/(\varepsilon-1)$, the implicitly chosen growth rate by the planner is lower than the market growth rate (remind that the planner chooses the first best number of firms, which is higher than the market supports). This reflects the predetermined weight on the lack of variety in the Dixit-Stiglitz specification, for which the planner solves.

Figure 2 compares all three growth trajectories with the accompanying variety for different values of σ.[17] The comparison of the SBSO with the FBSO shows, as stated above, that the optimal number of product varieties is equal in both cases. As the available instruments are limited in the SBSO it is obvious that, given the equality of the optimal variety, the growth rate always must be lower in the SBSO than in the FBSO. Taken that the optimal number of firms is equal in the two welfare exercises and that a the trade-off between growth and variety exists, the comparison between the market rate of growth and the growth rate attained by SBSO policy is obvious. It should be noted that with a permits policy it is impossible to improve both on the number of varieties and the growth rate at the same time.

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16 We refer to Appendix C for the solution of the growth rate.

17 The curve for the SBSO growth rate is drawn using the facts that $\partial g^{SB}/\partial\sigma<0$ and $\partial^2 g^{SB}/\partial\sigma^2>0$. We refer to Appendix C for a derivation.

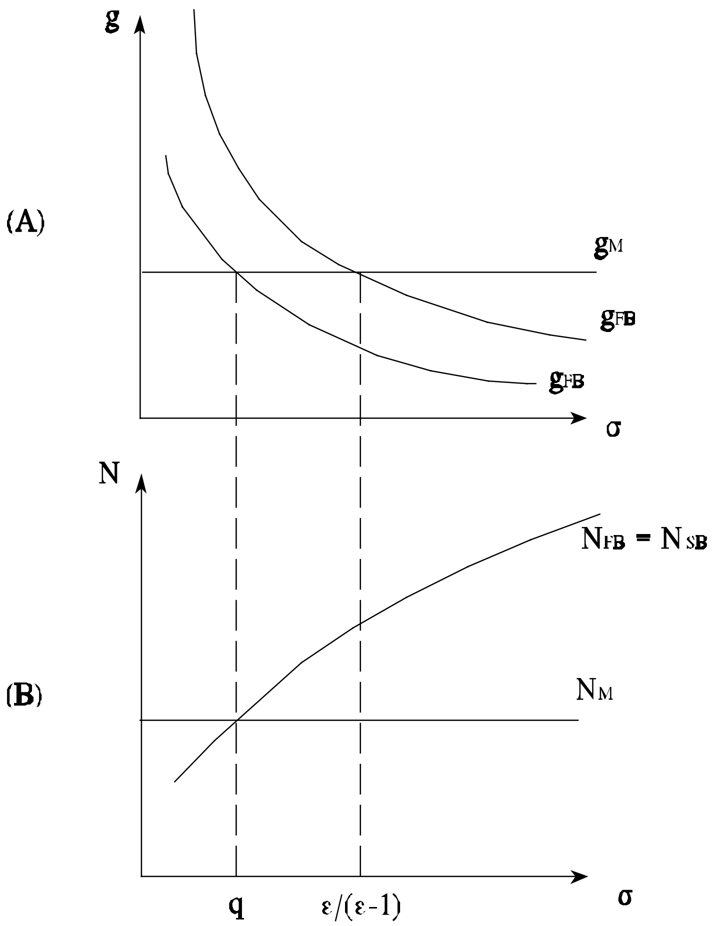

Figure 2. FBSO vs. SBSO

6. Conclusion

This paper has developed a simple two sector model of endogenous growth. Increased variety of intermediates was shown to have both a positive effect on welfare via increased productivity and negative effects via reduced growth and reduced availability of productive labour. Three potential market failures where distinguished, *i.e.*, a product diversity effect, a business stealing effect, and a pricing distortion. The market was shown not to provide in any case an equilibrium that reflects a socially desirable trade-off between variety and growth. When returns to diversity are strong relative to the toughness of competition, the market supports a too high growth rate and yields a lack of variety. When the returns to diversity are very weak this conclusion is reversed. The intermediate case might deliver a growth rate and a number of varieties that is too low from a social point of view. These results illustrate the importance of disentangling the returns to variety and the imperfect substitutability of brands when analyzing the welfare properties of models of monopolistic competition with endogenous growth.

References

Barro R.J., Sala-i-Martin X. (1995): *Economic Growth*, McGraw-Hill, New York.

Benassy J.P. (1996): "Taste for Variety and Optimum Production Patterns in Monopolistic Competition", *Economics Letters*, 52, 41-47.

Broer D.P., Heijdra B.J. (1996): "The Intergenerational Distribution Effects of the Investment Tax Credit under Monopolistic Competition", *OCfEB Research Memorandum 9603*, Rotterdam.

Dixit A.K., Stiglitz J.E. (1977): "Monopolistic Competition and Optimum Product Diversity", *American Economic Review*, 67, 297-308.

Ethier W.J. (1982): "National and International Returns to Scale in the Modern Theory of International Trade", *American Economic Review*, 72, 389-405.

Grossman G.M., Helpman E. (1991): *Innovation and Growth in the Global Economy*, MIT Press, Cambridge, MA.

Klundert Th. van de, Smulders S. (1997): "Growth, Competition and Welfare", *Scandinavian Journal of Economics*, 99, 99-118.

Mankiw N.G., Whinston M.D. (1986): "Free Entry and Social Inefficiency", *Rand Journal of Economics*, 17, 48-58.

Peretto P.F. (1996): "Sunk Costs, Market Structure and Growth", *International Economic Review*, 37, 895-923.

Romer P. (1990): "Endogenous Technological Change", *Journal of Political Economy*, 98, 71-102.

Spence M. (1976): "Product Selection, Fixed Costs, and Monopolistic Competition", *Review of Economic Studies*, 43, 217-235.

Yang X., Heijdra B.J. (1993): "Monopolistic Competition and Optimum Product Diversity: Comment", *American Economic Review*, 83, 295-301.

Appendix A. Solution of the Model

In this Appendix, we characterize the steady state solution of the model. Using the expressions for the price of intermediates (equation (13)) and the allocation rule of research labour (equation (14)), we can derive

$$\frac{\dot{p}_x}{p_x} = \frac{\dot{w}}{w} - \frac{\dot{h}}{h} = \frac{\dot{w}}{w} - g = \frac{\dot{p}_h}{p_h}. \tag{A.1}$$

Note that the wage rate is the numéraire (w=1). In the steady state, the allocation of labour is constant, and the number of firms is fixed. This gives rise to

$$g_C = \beta g_x = \beta g \quad \text{and} \quad \frac{\dot{P}_C}{P_C} = \beta \frac{\dot{p}_x}{p_x} = -\beta g. \tag{A.2}$$

So we can write the Ramsey rule as

$$\beta g = \frac{r + \beta g - \theta}{\rho}. \tag{A.3}$$

This equation corresponds to equation (18) in the main text.

Using equations (13), (14), and (A.1), the no-arbitrage condition (15) yields

$$-g - r = -\xi(L_x + L_r). \tag{A.4}$$

Imposing free entry and exit in the intermediate goods sector results in zero excess profits (equation (17)). Substituting the price for intermediates (equation (13)) and equation (9) into the zero profit condition (equation (17)), it boils down to

$$\frac{L_x + L_r + L_f}{L_x} = \frac{\varepsilon}{\varepsilon - 1}. \tag{A.5}$$

The firm size in relation to the size of the production department is thus equal to the mark-up. The larger the mark-up, the more a firm can afford to have large fixed costs (L_r+L_f) without making losses. The rate of return on investment can now be written as[18]

$$r = (\varepsilon - 1)(g + \xi L_f). \tag{A.6}$$

This corresponds to equation (18) in the main text.

Confronting the realized and required rates of returns yields the equilibrium growth and interest rate (equation (19) in the main text). The equilibrium number of high-tech firms and the allocation of labour can now be determined using labour market equilibrium (equation (16)), and

[18] Combining equations (A.4) and (10) results in L_x=r/ξ. Substitution of this expression for production labour, and L_r(=g/ξ) into equation (A.5) yields equation (A.6).

$$CP_C = \frac{Np_{xi}x_i}{\beta} = \frac{L_C w}{1-\beta} \quad \text{so} \quad \frac{NL_x \frac{\varepsilon}{\varepsilon-1}}{\beta} = \frac{L_C}{1-\beta}, \tag{A.7}$$

which shows how the assembly sector optimally chooses between labour and intermediates. This leaves us with

$$N = \frac{[\beta(\rho-1) - (\varepsilon-1)]\xi\beta L}{\varepsilon[\beta(\rho-1)\xi L_f - \theta]} \quad \text{and} \quad L_C = (1-\beta)L. \tag{A.8}$$

This equation corresponds to equation (20) in the main text.

Appendix B. The First Best Social Optimum

This Appendix solves for the optimal number of firms in the intermediate goods sector in the first best social optimum. Differentiation of equation (24) with respect to time yields

$$\frac{\dot{p}_h}{p_h} = (1-\rho)\frac{\dot{C}}{C} + \frac{\dot{N}}{N} - \frac{\dot{L}_C}{L_C} - \frac{\dot{h}}{h}. \tag{B.1}$$

We know by combining the equations (24) and (26) that

$$\frac{\dot{p}_h}{p_h} = \theta - \xi L_r - \frac{\beta\xi L_C}{N(1-\beta)}. \tag{B.2}$$

By differentiation of equation (3) with respect to time, we get (using symmetry among the intermediate goods producers)

$$\frac{\dot{C}}{C} = \beta\frac{\dot{X}}{X} + (1-\beta)\frac{\dot{L}_C}{L_C} = \beta\left[(\sigma-1)\frac{\dot{N}}{N} + \frac{\dot{N}}{N} + \frac{\dot{h}}{h} + \frac{\dot{L}_x}{L_x}\right] + (1-\beta)\frac{\dot{L}_C}{L_C}. \tag{B.3}$$

We now use the fact that L_C and NL_x are constant to arrive at

$$\frac{\dot{N}}{N}[1 + \beta(1-\rho)(\sigma-1)] = \theta - \frac{\beta\xi L_C}{N(1-\beta)} - \beta(1-\rho)\frac{\dot{h}}{h}. \tag{B.4}$$

Finally, using the solutions for L_C and the growth rate of h (equations (25) and (27), respectively), we arrive at the following differential equation in N

$$\dot{N} = \frac{\beta(\rho-1)\xi L_f - \theta}{\beta(\rho-1)(\sigma-1) - 1}N - \frac{\beta\xi L}{\beta(\sigma-1)+1}. \tag{B.5}$$

Under the restriction $1/(\beta(\rho-1))<\sigma-1<\xi L_f/\theta$ the root of the differential equation is positive and the optimum is characterized by a positive and constant number of firms. This optimum is obtained by setting $\dot{N}$ equal to zero. This yields equation (28) in the main text.

Appendix C. The Blocked Entry Model (*N* fixed).

This Appendix solves the model under the assumption that the number of firms is fixed (equation (A.5) is replaced by $N=\bar{N}$). The system of equations from which the solution can be found consists of equations (A.3), (A.4), (A.7), (10) (combined with the definition for the rate of growth), (16), and $N=\bar{N}$. We can solve this system of six equations with six unknowns (g, r, L_x, L_r, L_C, N).

The solution for the growth rate is

$$g = \frac{\xi \frac{L}{\bar{N}} - \xi L_f - \frac{\varepsilon - \beta}{\beta(\varepsilon - 1)}\theta}{(\rho - 1)\frac{\varepsilon - \beta}{\varepsilon - 1} + 1}. \tag{C.1}$$

Combining the Ramsey rule and the fact that $L_x = r/\xi$ (see footnote 18), we can derive

$$L_x = \beta(\rho - 1)\frac{\frac{L}{\bar{N}} - L_f + \frac{1}{\beta(\rho - 1)}\frac{\theta}{\xi}}{(\rho - 1)\frac{\varepsilon - \beta}{\varepsilon - 1} + 1}. \tag{C.2}$$

Finally, we derive labour used in the consumption goods sector as

$$L_C = (1 - \beta)\bar{N}\frac{\varepsilon}{\varepsilon - 1}(\rho - 1)\frac{\frac{L}{\bar{N}} - L_f + \frac{1}{\beta(\rho - 1)}\frac{\theta}{\xi}}{(\rho - 1)\frac{\varepsilon - \beta}{\varepsilon - 1} + 1}. \tag{C.3}$$

Taking derivatives yields

$$\frac{\partial L_x}{\partial \bar{N}} < 0; \quad \frac{\partial L_C}{\partial \bar{N}} < 0; \quad \frac{\partial g}{\partial \bar{N}} < 0. \tag{C.4}$$

The optimum is found by putting dU_0/dN equal to zero (see equation (34)) which results in

$$N^{SB} = \frac{(\beta(\rho - 1)(\sigma - 1) - 1)L}{[(1 - \beta) + \beta\sigma]\left[L_f(\rho - 1) - \frac{\theta}{\xi\beta}\right]}. \tag{C.5}$$

Substituting this solution for the second best number of firms into equation (44) yields the solution for the second best rate of growth

$$g^{SB} = (\varepsilon - 1)\frac{\xi L_f\left[\frac{\rho}{\beta(\rho - 1)(\sigma - 1) - 1}\right] - \theta\left[\frac{\beta\rho(\sigma - 1)(\varepsilon - \beta) - \beta(\sigma - 1)(1 - \beta) - (1 - \beta)}{\beta(\varepsilon - 1)(\beta(\rho - 1)(\sigma - 1) - 1)}\right]}{(\varepsilon - \beta)(\rho - 1) + (\varepsilon - 1)}. \tag{C.6}$$

Finally, we compare the growth rate in the second best social optimum with the first best rate of growth (equation (28)). This yields

$$g^{SB} - g^{FB} = \frac{(1-\beta)\left[\dfrac{\theta}{\beta} - \xi L_f(\rho-1)\right] - 2\theta(\sigma-1)[(\rho(\varepsilon-\beta)-(1-\beta)]}{[\beta(\rho-1)(\sigma-1)-1][(\varepsilon-\beta)(\rho-1)+(\varepsilon-1)]}. \tag{C.7}$$

Using the conditions $\beta(\rho\text{-}1)>(\varepsilon\text{-}1)>\theta/(\xi L_f)$ and $\beta(\rho\text{-}1)>1/(\varepsilon\text{-}1)>\theta/(\xi L_f)$, it is easily seen that the growth rate in the SBSO is always lower than in the FBSO. Equation (C.1) gives the market rate of growth as a function of the number of intermediate goods producers. Using the notion that $N^{SB}=N^{FB}=N^{M}$ at the point q (see footnote 11), it is evident that $g^{M}=g^{SB}$ at $\sigma=q$. When returns to diversity are relatively strong ($\sigma>q$), growth in the market is larger than growth in the FBSO, whereas the opposite holds if $\sigma<q$. Using $\partial g/\partial\overline{N}<0$ (see Equation (C.1)) and $\partial N^{SB}/\partial\sigma>0$, we derive $\partial g^{SB}/\partial\sigma<0$. Along the same procedure, it follows that $\partial^2 g^{SB}/\partial\sigma^2>0$.